Using the definition of the derivative, prove the derivative of $\cos x$. **SCORE: _____ / 15 PTS**

NOTE: You may use the values of the two special limits proved in class without reproving them, if you state the values of those limits.

$$\lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h} = \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h \to 0} \left((\cos x) \frac{\cos h - 1}{h} - (\sin x) \frac{\sin h}{h} \right)$$

$$= \lim_{h \to 0} \cos x \lim_{h \to 0} \frac{\cos h - 1}{h} - \lim_{h \to 0} \sin x \lim_{h \to 0} \frac{\sin h}{h}$$

$$= (\cos x)(0) - (\sin x)(1) = -\sin x$$

MULTIPLE CHOICE. Circle the correct answer. Show work to prove your answer is correct. **SCORE: _____ / 20 PTS**

Which of the following curves is orthogonal to the curve $y = \ln(\sec x - \tan x)$?

[a] $y = \sin x$ [b] $y = -\sin x$ [c] $y = \cos x$ [d] $y = -\cos x$ [e] $y = \cos x + \cot x$

$$\frac{dy}{dx} = \frac{1}{\sec x - \tan x}(\sec x \tan x - \sec^2 x)$$

$$= \frac{-\sec x \cancel{(\sec x - \tan x)}}{\cancel{\sec x - \tan x}}$$

OTHER $\frac{dy}{dx} = -\frac{1}{-\sec x} = \cos x$

If $f(x) = \dfrac{8x^2 - 12x}{\sqrt[4]{x}}$, find $\dfrac{d^3y}{dx^3}$. **SCORE: _____ / 15 PTS**

$$f(x) = 8x^{\frac{7}{4}} - 12x^{\frac{3}{4}}$$

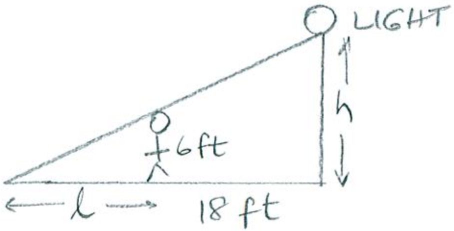
$$\frac{dy}{dx} = 14x^{\frac{3}{4}} - 9x^{-\frac{1}{4}}$$

$$\frac{d^2y}{dx^2} = \frac{21}{2}x^{-\frac{1}{4}} + \frac{9}{4}x^{-\frac{5}{4}}$$

$$\frac{d^3y}{dx^3} = -\frac{21}{8}x^{-\frac{5}{4}} - \frac{45}{16}x^{-\frac{9}{4}}$$

BJ, who is 6 ft tall, is standing 18 ft away from a floodlight sitting on a stage. If the floodlight is being pulled upward at 2 ft per second, how quickly is the length of BJ's shadow changing when the floodlight is 15 ft above the stage ? **SCORE: _____ / 25 PTS**

NOTE: Give the units of your final answer. State clearly whether BJ's shadow is growing or shrinking.



$$\frac{dh}{dt} = 2 \text{ ft/sec}$$

WANT $\left.\frac{dl}{dt}\right|_{h=15ft}$

$$\frac{l}{6ft} = \frac{l+18ft}{h}$$

$$hl = 6l\,ft + 108\,ft^2 \longrightarrow h = 15ft \quad 15l\,ft = 6l\,ft + 108\,ft^2$$

$$9l\,ft = 108\,ft^2$$

$$l = 12ft$$

$$l\frac{dh}{dt} + h\frac{dl}{dt} = 6ft\frac{dl}{dt}$$

$$(12ft)(2\tfrac{ft}{sec}) + (15ft)\frac{dl}{dt} = 6ft\frac{dl}{dt}$$

$$24\frac{ft^2}{sec} = -9ft\frac{dl}{dt}$$

$$\frac{dl}{dt} = -\frac{8}{3}\frac{ft}{sec}$$

BJ'S SHADOW IS SHRINKING

If $f(x) = (\cot x)^{\tan^{-1} x}$, find $f'(x)$.

SCORE: _____ / 25 PTS

$$\ln f(x) = \tan^{-1} x \ln \cot x$$

$$\frac{1}{f(x)} f'(x) = \frac{1}{1+x^2} \ln \cot x + \tan^{-1} x \frac{1}{\cot x} (-\csc^2 x)$$

$$f'(x) = (\cot x)^{\tan^{-1} x} \left(\frac{\ln \cot x}{1+x^2} - \frac{\tan^{-1} x \csc^2 x}{\cot x} \right)$$

Find the **equation of the normal line** to the curve $\cos^{-1}(5x+\frac{y^2}{2})=\frac{2\pi}{y}$ at $(-1,3)$. **SCORE: _____ / 25 PTS**

$$5x+\frac{y^2}{2}=\cos\frac{2\pi}{y}$$

$$5+y\frac{dy}{dx}=\left(-\sin\frac{2\pi}{y}\right)\left(-\frac{2\pi}{y^2}\frac{dy}{dx}\right)$$

$$5+3\left.\frac{dy}{dx}\right|_{(-1,3)}=\left(-\sin\frac{2\pi}{3}\right)\left(-\frac{2\pi}{9}\left.\frac{dy}{dx}\right|_{(-1,3)}\right)$$

$$5+3\left.\frac{dy}{dx}\right|_{(-1,3)}=-\frac{\sqrt{3}}{2}\cdot\frac{-2\pi}{9}\left.\frac{dy}{dx}\right|_{(-1,3)}$$

$$5=\left(\frac{\sqrt{3}\pi}{9}-3\right)\left.\frac{dy}{dx}\right|_{(-1,3)}$$

$$\left.\frac{dy}{dx}\right|_{(-1,3)}=\frac{5}{\frac{\sqrt{3}\pi}{9}-3}$$

$$y-3=\frac{3-\frac{\sqrt{3}\pi}{9}}{5}(x+1)$$

$$y-3=\frac{27-\pi\sqrt{3}}{45}(x+1)$$

Let $f(x) = \dfrac{e^{3-3x}}{5^x - 1}$.

SCORE: _____ / 25 PTS

[a] If x changes from 1 to 1.25, find dy.

$$\frac{dy}{dx} = \frac{-3e^{3-3x}(5^x-1) - e^{3-3x}(5^x \ln 5)}{(5^x-1)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{-3e^0(5^1-1) - e^0(5^1 \ln 5)}{(5^1-1)^2} = \frac{-12-5\ln 5}{16}$$

$$dy = \frac{-12-5\ln 5}{16}(1.25-1) = \frac{-12-5\ln 5}{64}$$

[b] Approximate $f(1.25)$.

$$y(1)+dy = \frac{e^0}{5^1-1} + \frac{-12-5\ln 5}{64}$$

$$= \frac{1}{4} + \frac{-12-5\ln 5}{64}$$

$$= \frac{4-5\ln 5}{64}$$